

Σ SCiM 2025



BOOK OF ABSTRACTS

16th European Symposium on Computational
Intelligence and Mathematics
May 18th – 21st, 2025 • A Coruña, Spain

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Book of abstracts of ESCIM 2025

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Program of the
16th European Symposium
on Computational Intelligence and Mathematics
including Workshop DigForASP
May 18th - 21st, 2025. A Coruña, Spain



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DIGFORASP



MONDAY 19 th	
Location: EXE Eurostarts Hotel	
8:30	Open Registration Desk
9:00–9:30	Inauguration
9:30–10:20	Keynote Speaker - Vladik Kreinovich Title: Spectacular Successes of Deep Learning: Why and What Next? Chairperson: Jesús Medina
10:20–11:20	Regular Session S1 Chairperson: Vladik Kreinovich
	<i>Virtual Assistant supported by Fusion of Fuzzy Rule-based Explainers</i> Pablo Pérez, Alejandro Catalá, Alberto Bugarín, José Alonso
	<i>Fuzzy Weighted Shapley Values based on Nearest Neighbor Classification</i> Marcin Ostrowski, Katarzyna Kaczmarek-Majer, Jan Budzianowski, Paweł Burchardt, Janusz Rzeźniczak
	<i>Logarithmic Number System Is Optimal for AI Computations: Theoretical Explanation of Empirical Success</i> Olga Kosheleva, Vladik Kreinovich, Christoph Lauter, Kristalys Ruiz-Rohena
11:20–11:40	Coffee break
11:40–13:20	Special Session: Homage to Alexander Šostak Chairperson: Olga Grigorenko
	<i>Relational Characterization of b-Metrics</i> Olga Grigorenko
	<i>On Application of Formal Fuzzy Contexts in Data Comparison Models</i> Maris Krastins
	<i>Solving linear equations using generalized equality</i> Branimir Šešelja, Andreja Tepavčević
	<i>Degree of validity of attribute implications based on multi-adjoint concept-forming</i> M. Eugenia Cornejo, Jesús Medina, Francisco J. Ocaña-Alcázar
	<i>(Ab) Comparison of Formal Concept Analysis and k-means Clustering on Healthcare Dataset</i> Štefan Puci, Matúš Semančík, Dominika Kotlářová, L'ubomír Antoni, Peter Eliaš, Ján Guniš, Stanislav Krajčí, Ondrej Krídlo, L'ubomir Šnajder
13:20–14:40	Lunch

14:40–15:30	Keynote Speaker - Marek Reformat Title: Category Theory in Action: A Novel Framework for Knowledge Graph Analysis and Reasoning. Chairperson: László T. Kóczy.
15:30–16:00	Coffee break
16:00–17:20	Regular Session S2 Chairperson: Marek Reformat
	<i>Decision-making based on confirmation measures</i> Fernando Chacón-Gómez, M. Eugenia Cornejo, Jesús Medina
	<i>Network Visualization of a Fuzzy Measure</i> Carlos I. Pérez-Sechi, Javier Castro, Daniel Gómez, Inmaculada Gutiérrez, Daniel Martín, Rosa Espínola
	<i>Complex-Valued Neural Networks: Applications and Performance in Image Classification</i> Piotr A. Kowalski, Michał Rutkowski
	<i>The granular structure of covering-based rough sets generated by the Mapper algorithm</i> Wannes De Maeyer, Mauricio Restrepo, Chris Cornelis
17:20–18:15	Regular Session S3 Chairperson: Radko Mesiar
	<i>Problem of distributivity between 2-uninorms and quasy-linear means</i> Dragan Jočić
	<i>Choquet-inspired functions</i> Humberto Bustince, Radko Mesiar, Gracaliz Dimuro, Bernard De Baets
	<i>(Ab) Unifunctions, a new class of functions for bipolar modeling</i> Juraj Kalafut, Martin Kalina
20:00	Welcome reception

TUESDAY 20 th	
Location: EXE Eurostarts Hotel	
9:00–9:50	Keynote Speaker - Pedro Cabalar Title: Normative Reasoning with Deontic Answer Set Programming. Chairperson: Stefania Costantini
9:50–10:50	Workshop DigForASP. Session 1 Chairperson: Pedro Cabalar
	<i>Formal Verification of Social Engineering in Information-Seeking Dialogues</i> Andreas Brännström, Juan Carlos Nieves
	<i>Blueprint Personas in Digital Forensics and Cybersecurity</i> Alina Vozna, Stefania Costantini, Andrea Monaldini, Raffaele Olivieri
	<i>(Ab) SM-based Semantics for Answer Set Programs Containing Conditional Literals and Arithmetic</i> Yuliya Lierler
10:50–11:10	Coffee break
11:10–12:00	Workshop DigForASP. Session 2 Chairperson: Juan Carlos Nieves
	<i>(Ab) Recursive Aggregates as Intensional Functions in Answer Set Programmin</i> Jorge Fandinno, Zachary Hansen
	<i>(Ab) Temporal Rules for Fraud Detection</i> Przemyslaw Walega
	<i>(Ab) Unfiltered Large Language Models: Forensic and Legal Challenges of their Criminal Use on the Dark Web. Preliminary Study</i> Dévika Pérez-Medina, Manuel Ojeda-Hernández, Fernando Chacón-Gómez
12:00–13:30	Special Session: Decision and optimization model applied to logistics and transport Chairperson: Julio Alberto López-Gómez
	<i>Time-Dependent Triangular Fuzzy Soft Set Model for Describing Traffic Dynamics</i> Boldizsár Tüü-Szabó, Ruba AlMahasneh, László T. Kóczy
	<i>Learning label ranking trees using evolutionary algorithms</i> Arturo Fernández-Mora, José Antonio Gámez, Juan Carlos Alfaro, Juan Moreno-García
	<i>(Ab) Fuzzy logic applied to decision and optimization models</i> David Muñoz Valero, Juan Moreno-García, Julio Alberto López-Gómez, Enrique Adrián Villarrubia-Martín
	<i>A multi-adjoint lattice logic based on Gödel logic</i> M. Eugenia Cornejo, Francesc Esteva, Luis Fariñas del Cerro, Lluís Godo, Jesús Medina
	<i>Relations Among Intermediate Syllogisms</i> Petra Murinová, Vilém Novak
13:30–15:00	Lunch
15:00–16:15	Regular Session S4 Chairperson: Manuel Ojeda-Aciego
	<i>On testing hypotheses in fuzzy surroundings</i> Aleksandar Takači, Ivana Štajner-Papuga, Zagorka Lozanov-Crvenković, Gabriјela Grujić, Tatjana Došenović, Dragan Jočić
	<i>Kolmogorov Representation Theorem Explained in Terms of Fuzzy Partitions</i> Irina Perfilieva
	<i>Systems of fuzzy relational equations in case of a complete codomain lattice</i> Vanja Stepanović, Andreja Tepavčević
	<i>(Ab) WAMs based on a pair of n-tuples (w, b) and their relationship to OWA and IOWA operators</i> Andrea Stupňanová, Radko Mesiar, Radomír Halaš, Jozef Pócz
17:00–20:30	A Coruña tour
20:30	Gala dinner

WEDNESDAY 21 st	
Location: EXE Eurostarts Hotel	
10:00–10:50	Keynote Speaker - Bernadette Bouchon-Meunier Title: Entropies, measures of uncertain and imprecise information. Chairperson: László T. Kóczy.
10:50–11:10	Coffee break
11:10–12:00	Regular Session S5 Chairperson: Bernadette Bouchon-Meunier
	<i>Analytical Properties of General Unary Hypotheses Automaton Quantifiers</i> Martina Daňková
	<i>(Ab) Linear spline copulas</i> Radko Mesiar, Andrea Stupňanová, Fateme Kouchakinejad
	<i>(Ab) Some notes on permutation-equivariant decomposition integrals</i> Adam Šeliga
12:00–13:40	Special Session: Recent trends in knowledge representation and modelling Chairperson: Roberto G. Aragón
	<i>Vertical sum of concept lattices via bonds</i> Roberto G. Aragón, Jesús Medina, Samuel Molina
	<i>Bipolar max-product fuzzy relation equations with floor negation</i> Antonio Castejón, David Lobo, Pablo López-Molina
	<i>(Ab) New Simplification Rules for Databases with Positive and Negative Attributes</i> Domingo López-Rodríguez, Manuel Ojeda-Hernández, Carlos Bejines
	<i>(Ab) Formal concept analysis and its fuzzy extensions for Light-Bot game solutions</i> Dominika Kotlářová, Peter Eliaš, Ján Guniš, L'ubomir Antoni, Stanislav Krajčí, Ondrej Krídlo, L'ubomír Šnajder
	<i>(Ab) Composition as a fuzzy conjunction between indexes of inclusion</i> Nicolás Madrid, Manuel Ojeda-Aciego
	<i>(Ab) Towards the use of the f-index of inclusion for fuzzy reasoning</i> Nicolás Madrid, Eloísa Ramírez-Poussa, Carolina Díaz-Montarroso
13:40–15:00	Closing Session

Social Events	
SUNDAY 18 th	
08:30	Tour to Santiago de Compostela
MONDAY 19 th	
20:00	Welcome reception
TUESDAY 20 th	
17:00-20:30	A Coruña tour
20:30	Gala dinner
WEDNESDAY 21 st	
13:40-15:00	Closing Session

Keynote speech:

Spectacular Successes of Deep Learning: Why and What Next?

Vladik Kreinovich

Department of Computer Science, University of Texas,
El Paso, United States



Abstract: Successes of deep learning are partly due to appropriate selection of activation function, pooling functions, etc. Most of these choices have been made based on empirical comparison and heuristic ideas. In the first – why — part of this talk, we show that many of these choices – and the surprising success of deep learning in the first place – can be explained by reasonably simple and natural mathematics.

In the second – what next — part, we present ideas on how to overcome limitations of current deep learning techniques. One of these limitations is that, in contrast to humans who can learn from a few examples and learn fast, modern deep learning techniques require a large amount of data to learn, and they take a long time to train. In this talk, we show that neural networks do have a potential to learn from a small number of examples – and learn fast. We speculate that the corresponding idea may already be implicitly implemented in Large Language Models – which may partially explain their (somewhat mysterious) success.

Virtual Assistant supported by Fusion of Fuzzy Rule-based Explainers

Pablo Miguel Pérez-Ferreiro, Alejandro Catala, Alberto Bugarín-Diz and Jose M. Alonso-Mora

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Abstract: In this work, we introduce a novel methodology for combining different explanatory inputs into a singular coherent narrative in the medical domain. As proof of concept, we have implemented a prototype ready to diagnose diabetes and heart disease. The proposal is validated with a user study in which we collected 48 valid responses to a given questionnaire. We compared four different strategies for explanation fusion, considering explanations coming from two different fuzzy rule bases. The winner strategy (i.e., verbalizing common antecedents as the main factors of the diagnosis and adding any non-common antecedents as supporting evidence) was perceived as trustworthy, useful, and natural by most participants in the study.

Keywords: Explainable Fuzzy Systems · Natural Language Generation · Trustworthy Conversational Agents · Human Evaluation.

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Fuzzy Weighted Shapley Values Based on Nearest Neighbor Classification

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Abstract: In this study, we propose a novel modification of the Exact Shapley Values Based on Nearest Neighbor Classification method that incorporates fuzzy logic to better account for uncertainty in datasets. The modification introduces a fuzzy weight vector computed using the Fuzzy K Nearest Neighbor algorithm, which improves the computation of Shapley values for pseudo-labeled instances. This approach aims to better reflect the relevance of observations and mitigate the effects of data uncertainty. The method was validated on a real-world pilot study of 654 patients diagnosed with acute coronary syndrome. In the dataset used, one of the problems encountered in clinical practice, data uncertainty, emerged. The results show that the K-FVSNN method achieves competitive performance and maintains robust results even with up to 90% missing labels in the training set. These results highlight the potential of the K-FVSNN method for handling uncertain data in medical applications. Future work will explore its application to other datasets and further refinements to the weight vector to improve its generalizability.

Keywords: Shapley Values · semi-supervised learning · classification · missing data · medical data.

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Logarithmic Number System Is Optimal for AI Computations: Theoretical Explanation of an Empirical Success

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Abstract: Current AI systems are very successful, but they are not perfect, they need to be trained better. Training modern AI systems requires a tremendous amount of computations – that already take a lot of time. To increase the number of computations, we need to make each computation step faster. It was shown that we can speed up computations if we apply an appropriate nonlinear transformation to all the values, and that logarithmic transformation leads to the fastest speedup. In this paper, we provide a theoretical explanation for this empirical success.

Keywords: Machine learning · AI · Nonlinear transformations · Logarithmic transformation · Optimal transformation.

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Relational Characterization of b-Metrics

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Abstract: This paper investigates the concept of b-metrics through relational structures. In particular, we examine commonly used definitions of b-metrics and establish a unique characterization of b-metrics in terms of linear order relations. Our findings provide a deeper understanding of the properties of b-metrics and their underlying relational framework.

Keywords: B-Metric · Metric · Fuzzy Metric.

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On Application of Formal Fuzzy Contexts in Data Comparison Models

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Abstract: Noticing the problem of precise matching of fuzzy sets of objects and fuzzy sets of properties in fuzzy concept analysis and the difficulties caused by it for practical applications, we propose to replace concept lattices by more flexible, namely graded *fuzzy preconcept lattices*. Based on fuzzy preconcept lattices, we develop the foundations of the graded approach for all three versions of fuzzy concept analysis: the “classical” formal concept analysis, the object-oriented concept analysis and the property-oriented concept analysis. This has been one of the last researches carried out by professor Alexander Šostak who developed the main theoretical concepts which were further applied in practical studies by both authors.

The notion of a formal context as the basis for formal concept analysis first appeared in papers by R. Wille and B. Ganter in early 80-ties of the previous century and later developed into a full-bodied theory (see, e.g. [1] and [2]) having numerous important applications. A formal context is a triple (X, Y, R) where X, Y are non-empty sets and $R \subseteq X \times Y$ is a relation. The usual interpretation of a context (X, Y, R) is that X is the set of some abstract objects, Y is the set of some abstract properties or attributes and the entry $(x, y) \in R$ abbreviated as xRy means that an object x has the attribute y . The idea of concept analysis is to distinguish those pairs $(A, B) \in 2^X \times 2^Y$ called (formal) concepts which are in a specific way mutually connected via relation R . Specifically, according to R. Wille and B. Ganter a formal concept in a context (X, Y, R) is a pair (A, B) such that a property y is in B if and only if all $x \in A$ have property y (i.e. are related to y) and conversely, an object x is in A if and only if it has all the properties $y \in B$ (i.e. all $y \in B$ are related to x). The concept as defined by R. Wille and B. Ganter is not the only one natural way to unite sets of objects and sets of attributes in reasonable pairs of “*concepts*” by means of the relation $R \subset X \times Y$. In particular I. Düntch and G. Gediga [3], working in the field of modal logic, suggested to consider as concepts the pairs of objects and attributes $A \subseteq X, B \subseteq Y$ such that a property y is in B if and only if all objects having these properties are contained in A and an object x is in A if and only if it has at least one property $y \in B$.

In definition of a concept in a context (X, Y, R) proposed by R. Wille and B.

Ganter, the set X of objects play the same role as the set Y of attributes: if one renames the elements of X as attributes and the elements of Y as objects the pairs (A, B) presenting concepts remain the same. In other words, the context (X, Y, R) is equivalent to the context (Y, X, R) . On the other hand, in definition of a context (X, Y, R) proposed by I. Düntch and G. Gediga the roles of the set X of objects and the set Y of attributes are different: a concept (A, B) in the context (X, Y, R) becomes a concept in the context (Y, X, R^{-1}) . This observation prompted Y. Y. Yao [4] to present an approach which is dual to the approach used by I. Düntch and G. Gediga for comprehension of a concept in the context (X, Y, R) . It is usually referred now as object-oriented as different from attribute oriented to the interpretation of a concept in a context (X, Y, R) . To develop a fuzzy version of a formal concept analysis one naturally have to start with a fuzzy version of a formal context, that is a quadruple (X, Y, L, R) where X, Y are sets L is a complete lattice, probably with an additional algebraic structure and $R : X \times Y \rightarrow L$ is an L -fuzzy relation. The fuzzy version of the concept of R. Wille and B. Ganter was introduced by R. Belohlavek in [5] while fuzzy versions of concepts introduced by I. Düntch and G. Gediga as well as Y. Y. Yao first appeared as far as we know in [6].

In our research we replace fuzzy concepts as the focus of the study by a very general notion of a preconcept viewing it as a certain “potential concept”. Further, by means of fuzzy logic tools we prescribe to each preconcept a grade, that is the measure of its “conceptuality”. In the result we obtain a graded fuzzy preconcept lattice. The graded fuzzy preconcept lattices on the lines of the classical, i.e. concept analysis of R. Wille and B. Ganter were first considered in our paper [7]. In turn, graded fuzzy concepts in the spirits of oriented concept analysis were exposed in our talk at the CLA2022 conference [8]. It is the goal of this work to incorporate further to the graded versions of all three “branches” of fuzzy concept analysis, to compare them and to illustrate the scope of their applications in data comparison models.

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Solving linear equations using generalized equality

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Abstract: We present a method of solving linear equations over a structure with a binary operation, in the situations of missing or corrupted (unclear) data. For this purpose we use the framework of poset-valued functions, replacing the classical equality by a symmetric and transitive map from the given structure of data into the suitable poset. This generalized equality is additionally assumed to be compatible with the given binary operation. Our technique can be considered as a new way of solving linear equations if the classical technique is not applicable.

Keywords: P-sets · generalized equality · P-quasigroups · approximate solutions of equations.

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Degree of validity of attribute implications based on multi-adjoint concept-forming operators

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Abstract: The study of attribute implications in fuzzy settings plays a crucial role in knowledge representation and data analysis with vague or imprecise information. In this paper, we propose a new definition of validity of an attribute implication within the fuzzy framework of multi-adjoint concept lattices, taking into account the original definition in terms of the derivation operators.

Keywords: Attribute implications · Validity · Multi-adjoint concept lattices

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Comparison of Formal Concept Analysis and k -means Clustering on Healthcare Dataset

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Abstract: The modified Rice-Siff algorithm [1,2] within the framework of (Fuzzy) Formal Concept Analysis [3,4,5,6] represents a method for efficient computation of formal concepts. This algorithm builds upon the foundational Rice-Siff approach and the modification incorporates optimizations to improve scalability and computational efficiency when applied to larger and more complex datasets. By modifying the formal concept generation process, the algorithm reduces redundancy and uses the structural properties of concept lattices. This enhancement is particularly relevant for applications in data analysis, knowledge representation, and other scientific domains [7,8]. Other interesting and powerful methods of fuzzification in Formal concept analysis were developed by various research groups [9,10,11,12,13].

The k -means clustering algorithm, one of the most widely used unsupervised machine learning algorithms for partitioning a dataset into distinct clusters based on similarity, was first proposed by Stuart Lloyd in 1957. Independently, the method was later investigated by James MacQueen in 1967, who denoted the term k -means and described its statistical properties in a clustering context [14]. The algorithm is based on minimizing intra-cluster variance while maximizing inter-cluster distance, providing a grouping of data points. Despite its interpretability and computational efficiency, k -Means has limitations, such as sensitivity to the initial selection of centroids and difficulty handling clusters of varying sizes or densities. It was applied in various fields, including image segmentation, customer segmentation, or bioinformatics [15].

In this paper, we compare the modified Rice-Siff algorithm and k -means clustering on the publicly available dataset collected by the consortium I-CARE Cardiac Arrest REsearch [16]. Seven hospitals across the United States and Europe participated in this study, including two from The Netherlands, one from

Belgium, and three from Boston and New Haven, USA. The dataset comprises metadata and EEG recordings from 607 patients. Each patient contributed multiple EEG files, reflecting variations in the duration of monitoring. These recordings were obtained using a 19-channel longitudinal EEG setup. We constructed a tabular dataset from the metadata and our extracted features of EEG recordings in tabular form. Thus, we applied the methods of the modified Rice-Siff algorithm and k -means clustering on the constructed dataset and compared the similarity of the obtained clusters of patients. In summary, we present the differences between both approaches (e.g., modified Rice-Siff can generate overlapping clusters) and their advantages and formulate recommendations for data analysts.

Keywords: Formal Concept Analysis · Rice-Siff algorithm · k -means clustering · Healthcare are

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Keynote speech:

Category Theory in Action: A Novel Framework for Knowledge Graph Analysis and Reasoning

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Abstract: Knowledge graphs are suitable for modeling semantically rich data, but their analytical potential is unexplored. This talk describes a formal mathematical framework that employs category theory to reason about and analyze knowledge graphs at different levels of abstraction.

Using an explanatory academic collaboration graph, we illustrate how category notions like coverings, sieves, and Grothendieck topologies can reinforce different contextual viewpoints of a knowledge graph, enabling multi-perspective reasoning about entities and their relationships. We also illustrate how sheaves and co-sheaves provide mechanisms for data analysis and its cohesion across the graph structure.

Our approach enables a novel approach to analysis, including context-dependent reasoning, multi-scale reasoning, and local-to-global formalization of information flow.

Decision-making based on confirmation measures

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Abstract: Confirmation measures have been introduced in Rough Set Theory to analyze the influence of the antecedent on the consequent of decision rules. This paper extends these measures to the fuzzy setting and proposes their application in data classification to address decision-making problems. Furthermore, a particular confirmation measure is defined, studied in detail and applied to a real case.

Keywords: Fuzzy Rough Set Theory · Decision Rules · Confirmation Measures · Data Classification.

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Network Visualization of a Fuzzy Measure

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Abstract: This conference paper proposes a visualization method of fuzzy measures to model complex interactions between elements in dynamic environments. Despite the utility of fuzzy measures, they present challenges due to the complexity of their interpretation when the number of elements rises. We suggest using graphical representations of the Shapley Index and the Murofushi-Grabisch interaction index, to provide a more intuitive understanding of fuzzy measures. This approach enables faster and smarter decisions by simplifying the analysis of elements interaction.

Keywords: Decision Modeling · Soft Computing · Fuzzy Measures · Social Networks · Graph Theory

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Complex-Valued Neural Networks: Applications and Performance in Image Classification

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Abstract: This paper investigates the use of complex-valued neural networks in the image classification task, using the MNIST dataset as a case study. While real-valued neural networks have been extensively applied in various machine learning problems, complex-valued networks offer an alternative approach that can better model certain types of data, particularly in domains where oscillatory or periodic patterns are prevalent. We explore the theoretical foundations of complex-valued networks, highlighting their key differences from real-valued networks, particularly in the handling of input data and the computation of gradients during backpropagation. The study presents an experiment comparing the performance of complex-valued and real-valued networks for image classification, with the input data transformed into the complex domain using a discrete Fourier transform. The results demonstrate that complex-valued networks can achieve competitive performance, with faster convergence and higher accuracy in early training epochs, particularly when using the Adam optimization algorithm. This research provides insights into the potential advantages of complex-valued neural networks and their applicability to tasks involving complex data, suggesting new directions for future work in this area.

Keywords: Complex-Valued Neural Networks · Complex-Valued transfer function · image classification · MNIST dataset.

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The granular structure of covering-based rough sets generated by the Mapper algorithm

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Abstract: Covering-based rough sets generalize the notion of rough sets by changing the partition of a universe to a covering. In earlier work, the use of the Mapper algorithm (which was originally designed to visualize high dimensional datasets) to construct coverings of datasets was introduced. In this paper we combine Mapper and an adapted version of the DBSCAN clustering algorithm and prove that the generated coverings maintain the same granular structure as the lower-dimensional coverings that are used as input for Mapper. In particular, we investigate the case where the associated order between coverings is linear.

Keywords: Granular structure · Covering-based rough sets · Mapper algorithm

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Problem of distributivity between 2-uninorms and quasy-linear means

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Abstract: Distributivity laws between certain classes of aggregation operators play a fundamental role in a large number of disciplines such as fuzzy sets and fuzzy logic, pseudo-analysis and measure theory, and particularly in the utility theory. This paper is focused on the problem of distributivity between some classes of 2-uninorms and quasi-linear means.

Keywords: Uninorms · 2-uninorms · quasi-linear means · distributivity equations

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Unifunctions, a new class of functions for bipolar modeling

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Abstract: Uninorms [3], which were proposed as a common basis for t-norms and t-conorms, found their application in various areas. One of the reasons is their bipolar behavior, i.e., uninorms are strongly conjunctive below the neutral element, disjunctive above, and averaging otherwise. However, the axioms of uninorms imply that there is no uninorm that is continuous on the entire unit square, which leads to several problems that must be overcome, namely computational stability and the inability to make gradual changes.

In [1] Bustince et al. defined overlap functions as a parallel structure to t-norms. In this definition they impose continuity rather than associativity, arguing that in many cases of application associativity may be superfluous, but continuity is not. One of these cases is when the function is strictly binary and thus the n-ary form is unnecessary. Similarly, grouping functions [2] have been defined to replace t-conorms in an analogical way.

Following these steps, in this paper we want to introduce a new class of binary functions, called unifunctions, which on the one hand resemble uninorms as much as possible and on the other hand preserve continuity on the whole unit square. Since every uninorm is composed of a t-norm and a t-conorm, we want to preserve such a connection between a unifunction on the one side and an overlap and a grouping function on the other side. Therefore, the definition of a unifunction is as follows.

Definition 1. Let $e \in]0, 1[$. A binary function $UF_e : [0, 1]^2 \rightarrow [0, 1]$ is called a *unifunction* if the following conditions hold.

1. UF_e is non-decreasing.
2. UF_e is symmetric.
3. For $(x, y) \in [0, e]^2$, $UF_e(x, y) = 0$ if and only if $x = 0$ or $y = 0$.
4. For $(x, y) \in [e, 1]^2$, $UF_e(x, y) = 1$ if and only if $x = 1$ or $y = 1$.
5. $UF_e(x, y) = e$ then $x < e < y$ or $y < e < x$ or $x = y = e$.
6. UF_e is continuous

Although none of the unifunctions is associative, several interesting properties can be established, such as 1-Lipschitz, self-duality, and others. Note that a unifunction restricted to

- $[0, e]^2$ is isomorphic to an overlap function,
- $[e, 1]^2$ is isomorphic to a grouping function,

- $[0, e] \times [e, 1]$ is isomorphic to a continuous bipolar function.

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Keynote speech:

Normative Reasoning with Deontic Answer Set Programming

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Abstract: A rigorous verification of the compliance of Artificial Intelligence (AI) systems with social, ethical and legal regulations requires, in the first place, the design of formal languages for an accurate specification of norms. Normative reasoning falls under the domain of deontic logic, that studies the formalisation of concepts such as obligations, permissions and violations. In this talk, we will review a recent approach to normative reasoning based on a deontic extension of Answer Set Programming (ASP), a successful paradigm for practical Knowledge Representation and problem solving. To this aim, we will start reviewing the extension of Equilibrium Logic (the logical basis of ASP) to cope with deontic operators and their combination with explicit negation. Then, we will illustrate the use of this approach by formalising several challenging examples from the literature, implementing them on the deontic ASP tool «deolingo». Finally, we will move to consider temporal normative reasoning and the kind of concepts that arise when we combine deontic operators with time, explaining their recent formalisation based on Temporal Deontic Equilibrium Logic.

Formal Verification of Social Engineering in Information-Seeking Dialogues

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Abstract: In this paper, we apply formal dialogue methods to recognize and analyze phishing in interactions. Phishing attacks exploit human vulnerabilities, typically through deceptive and manipulative messages, leading victims to disclose sensitive information. Existing machine learning-based detection methods often lack transparency, making it difficult to trace manipulation tactics. We utilize the so-called Goal-Hiding Dialogue (GHD) framework, originally designed for reasoning about non-collaborative agents in information-seeking dialogues. The framework employs Quantitative Bipolar Argumentation Frameworks (QBAFs) to model how a seeker agent strategically influences a target’s willingness to engage with certain topics. Our approach provides a mathematically grounded method for identifying key conversational shifts where manipulation occurs, contributing to phishing detection and evidence analysis.

Keywords: Formal dialogues · Quantitative argumentation · Social engineering · Phishing · Information extraction · Non-collaborative agents

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Blueprint Personas in Digital Forensics and Cybersecurity

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Abstract: As cyber threats become increasingly sophisticated, traditional security measures struggle to adapt. This paper explores the integration of Blueprint Personas, a structured approach to modeling cyber threats, into AI-driven digital forensics and cybersecurity. By simulating adversary behaviors, these personas enhance threat intelligence, adversary emulation, and forensic investigations. Additionally, we introduce the Reference Ontology of Trust (ROT) to improve explainability and adaptive decision-making in AI-driven security systems. We also propose Intelligent Agents to automate threat detection and forensic analysis within Security Operations Centers (SOCs). This work-in-progress lays the foundation for a multi-agent security framework, aiming to enhance proactive cyber defense and real-time forensic automation.

Keywords: Intelligent Agent · Healthcare · Blueprint Personas · Ontology · Trust.

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SM-based Semantics for Answer Set Programs Containing Conditional Literals and Arithmetic

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Abstract: Answer Set Programming (ASP) is a declarative programming paradigm that has been applied within a variety of challenging and high-consequence systems such as explainable donor-patient matching and space shuttle decision support systems. ASP programs are concise, human-readable, and benefit from well-defined semantics rooted in mathematical logic – these qualities make ASP programs attractive candidates for formal verification. Providing high levels of assurance regarding program behavior is particularly crucial for safety-critical applications. This work is part of a research stream with the long-term goal of supporting rigorous verification of ASP-based systems.

Modern answer set programming solvers such as CLINGO support advanced language constructs that improve the expressivity and conciseness of logic programs. Conditional literals are one such construct. They form “subformulas” that behave as nested implications within the bodies of logic rules. Their inclusion brings the form of rules closer to the less restrictive syntax of first-order logic. Rules with conditional literals concisely express knowledge that may be difficult to otherwise encode. For instance, conditional literals are widely employed in meta-programming – Listings 4-7 in “How to build your own ASP-based system?!” by Kaminski et al. (2021). These listings define meta encodings which compute the classical and supported models of reified logic programs; these encodings rely heavily on conditional literals. Conditional literals may also make programs easier to formally verify by reducing the number of auxiliary or inessential predicates in a program.

Typically, the semantics of ASP programs with variables are defined indirectly, via a procedure called *grounding*. Grounding turns a given program with variables into a propositional one. Then, semantics are defined for the resulting propositional program. In 2011, Ferraris, Lee, and Lifschitz proposed semantics for ASP programs that bypasses grounding. They introduced the SM operator, which turns a program (or, rather, the first-order logic formula associated with the considered program) into a classical second-order formula. The Herbrand models of this formula coincide with the answer sets of the original program. In this project, we introduce grounding-free SM operator-based semantics for logic programs containing *both* conditional literals and arithmetic. This enhances our ability to reason about the behavior of programs independently of a specific grounding context. The established semantics for programs with conditional literals relies on a translation to non-classical infinitary propositional logic. We also establish the precise correspondence between the proposed and existing semantics.

Recursive Aggregates as Intensional Functions in Answer Set Programming

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Abstract: This paper shows that the semantics of programs with aggregates implemented by the solvers `clingo` and `dlv` can be characterized as extended First-Order formulas with intensional functions in the logic of Here-and-There. Furthermore, this characterization can be used to study the strong equivalence of programs with aggregates under either semantics. We also present a transformation that reduces the task of checking strong equivalence to reasoning in classical First-Order logic, which serves as a foundation for automating this procedure.

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Temporal Rules for Fraud Detection

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Abstract: Temporal logic programming provides a range of rule-based languages tailored for reasoning about temporal properties. These include various extensions of Datalog, such as Datalog_{1S}, which introduces an additional temporal sort and a single successor operator, and Datalog_{nS}, which generalizes this with n -ary functions for the temporal sort. Other examples include TempLog, which incorporates operators from linear temporal logic (LTL), and DatalogMTL, which leverages operators from metric temporal logic (MTL). Many of these languages support non-monotonic negation, as seen in temporal extensions of answer set programming with LTL or MTL operators, the stream reasoning language LARS, and non-monotonic extensions of DatalogMTL. Their theoretical properties, such as expressive power and computational complexity, are often well-studied, and efficient reasoning algorithms have already been implemented.

Temporal aspects of financial transactions play a vital role in detecting fraudulent activities, making temporal logic programming a valuable tool for identifying suspicious behaviors. The traditional approach involves experts manually crafting temporal rules. These rules are then applied to datasets of financial transactions, allowing to flag fraudulent accounts via logical reasoning. While this method is fully interpretable and explainable, it has significant limitations. Fraud detection rules are often complex and evolve over time, making their formulation both labor-intensive and prone to human error.

We propose two approaches to address these challenges. The first approach leverages rule-learning techniques to automatically extract fraud detection rules from temporal datasets. Recent advancements in rule learning, particularly in inductive logic programming, indicate that data-driven rule extraction can be successfully applied in various real world problems. The second approach involves training black-box machine learning models for fraud detection and subsequently explaining their decisions using temporal rules. Graph neural networks (GNNs) are particularly promising for this task due to their strong connections with logic and the growing area of temporal graph learning (TGL). During the talk we will introduce our first results obtained on both learning temporal rules from data [1] and on temporal graph neural networks [2].

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Unfiltered Large Language Models: Forensic and Legal Challenges of their Criminal Use on the Dark Web. Preliminary Study

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Abstract: The rapid development of large language models (LLMs) has revolutionised natural language processing and automatic text generation across all sectors. However, the emergence and spread of unfiltered or ethically unrestricted versions of these models has introduced new threats, especially in criminal ecosystems such as the Dark Web. These LLMs, devoid of alignment constraints, are being exploited to facilitate a wide range of illicit activities.

Among the most concerning tools is OnlyFake, highlighted in Europol's IOCTA 2024 report. This platform leverages generative AI to produce high-quality fake identities and documents at scale, posing a serious risk to identity verification systems and legal processes. In addition, models such as WormGPT, FraudGPT, and DarkGPT are being marketed on underground forums to automate phishing attacks, generate undetectable malware, and facilitate large-scale fraud and identity theft, even for users with limited technical expertise.

This presentation will examine the criminal use of unfiltered LLMs from two interrelated perspectives: computer forensics and criminal law. On the one hand, from a technical forensic point of view, the main tools, technical difficulties in detecting AI-generated content and identifying the use of generative models in criminal workflows will be described. From a criminal law perspective, the challenges of attributing criminal responsibility to developers, facilitators and users will be addressed, and a critical assessment will be made of whether existing criminal frameworks are sufficient to address the specificities of AI-facilitated crimes and the future challenges for criminal police investigation.

Keywords: Artificial Intelligence · Large Language Model · Digital Forensics · Dark Web · Deepfake

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Time-Dependent Triangular Fuzzy Soft Set Model for Describing Traffic Dynamics

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Abstract: Traffic route optimization in dynamic urban environments is challenging due to uncertainties such as congestion and fluctuating conditions during rush hours. This paper introduces a novel route optimization framework utilizing Triangular Fuzzy Numbers (TFNs) to model the time-dependent membership of road segments to traffic scenarios like jam regions and rush hour periods adapts to historical data and can integrate real-time updates for robust evaluations of traffic conditions. By integrating with time-dependent Traveling Salesman Problem (TDTSP) algorithms, the framework provides practical solutions for urban traffic management, improving route efficiency while accounting for real-world uncertainties.

Keywords: Route Optimization · Triangular Fuzzy Numbers · Fuzzy Soft Sets.

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Learning label ranking trees using evolutionary algorithms

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Abstract: This work presents a study on the application of evolutionary methods for generating label ranking trees. The goal is to order a set of classes based on their relevance or preference for each instance in the dataset. Throughout the study, a model based on evolutionary algorithms is implemented, the advantages and challenges of using evolutionary algorithms in this problem are analyzed and its performance evaluated in terms of accuracy and efficiency compared to classical approaches. Furthermore, an improvement is proposed by seeding the initial population by using the tree learned by using the greedy Label Ranking Tree algorithm. The results obtained demonstrate that the application of evolutionary algorithms to address label ranking problems gives an improvement to Label Ranking Tree in most of the datasets.

Keywords: label ranking · tree induction · evolutionary computation

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A multi-adjoint lattice logic based on Gödel logic

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Abstract: Multi-adjoint lattice logic (MLL) was introduced as a logic focused on capturing multi-adjoint algebras, which are general and flexible algebraic structures used, for example, as truth-values set in different formal tools to model data sets, such as, formal concept analysis, rough sets, and fuzzy relation equations. This paper enriches this logic by expanding it with two extra connectives, one associated with the Gödel implication and the one associated with the Baaz-Monteiro projection connective. As a consequence, the implication in MLL representing the ordering in the lattice becomes definable from these two operators, and vice versa.

Keywords: Bounded poset · multi-adjoint algebra · multi-adjoint lattice logic · fuzzy logic · Gödel fuzzy logic · Baaz-Monteiro operator

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Relations Among Intermediate Syllogisms

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Abstract: In this article, we will show how relations forming the graded Peterson's square of opposition (contrary, contradictory, sub-contrary and sub-alterns) are connected with intermediate logical syllogisms, i.e., syllogisms that contain intermediate quantifiers. Our results are related to the theory of non-monotonic reasoning.

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On testing hypotheses in fuzzy surroundings

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Abstract: Various statistical models have found their place in practical applications; however, working with fuzzy-valued data still presents a challenge. This paper provides an overview of several well-known concepts of statistical hypothesis testing in fuzzy environments and introduces a new approach based on the comparison of triangular fuzzy numbers along the horizontal axis, i.e., on a horizontal fuzzy relation. The proposed method allows both the registered value of the test statistic and the reference critical value to be in fuzzy form and defines fuzzy degree of acceptance of hypotheses.

Keywords: Fuzzy sets · fuzzy numbers · fuzzy relations · comparing fuzzy values · hypotheses testing

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Kolmogorov Representation Theorem Explained in Terms of Fuzzy Partitions

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Abstract: The main goal of this contribution is to show connection between the classical Kolmogorov-Arnold representation theorem and theoretical foundations of fuzzy modeling. Especially, we show that the class of inner functions in the Kolmogorov representation of a continuous multivariable function belongs, up to multiplicative constants, to the class of membership functions that constitute a fuzzy partition with the Ruspini condition.

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Systems of fuzzy relational equations in case of a complete codomain lattice

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Abstract: The paper deals with fuzzy relational equations in the case where fuzzy relations are defined as arbitrary mappings from the square of a domain set to a lattice. That lattice is usually a sort of special, residual, Heyting, Brouwerian, continuous etc. We consider the case of a complete codomain lattice, which is more general. The completeness of the codomain lattice implies the completeness of the lattice of all fuzzy relations on a set and of all its powers. Using Tarski fixed point theorem for an operator on a complete lattice, we prove the existence of a solution for a wide class of fuzzy relational equations and their systems. By doing this we generalize our previous results. Tarski theorem also implies the existence of the greatest and the least solution for the mentioned class of equations and their systems, and the fact that the sets of their solutions are complete lattices.

Keywords: Complete lattice · Fuzzy relational equations · Solvability · Extremal solutions

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WAMs based on a pair of n -tuples (\mathbf{w}, \mathbf{b}) and their relationship to OWA and IOWA operators

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Abstract: The OWA and IOWA operators are both based on a given weighting vector $\mathbf{w} \in [0, 1]^n$ and an ordering method expressed by appropriate permutations $\sigma : \{1, \dots, n\} \rightarrow \{1, \dots, n\}$. We shall denote Ω as the set of all permutations. In the case of the OWA, the weighting vector is determined by the input n -tuple \mathbf{x} itself. The OWA operator can be introduced by the following equation:

$$\begin{aligned} \text{OWA}_{\mathbf{w}}(\mathbf{x}) &= \text{AM} \left(\sum_{i=1}^n w_i x_{\sigma(i)} \mid \sigma \in \Omega, x_{\sigma(1)} \geq \dots \geq x_{\sigma(n)} \right) \\ &= \text{AM} \left(\sum_{i=1}^n w_{\sigma^{-1}(i)} x_i \mid \sigma \in \Omega, x_{\sigma(1)} \geq \dots \geq x_{\sigma(n)} \right), \end{aligned}$$

where AM is the arithmetic mean. In other words, the OWA operator is simply a weighted arithmetic mean, $\text{OWA}_{\mathbf{w}}(\mathbf{x}) = \text{WAM}_{\mathbf{w}, \mathbf{x}}(\mathbf{x})$, where

$$\mathbf{w}_{\mathbf{x}} = \text{AM}(\mathbf{w}_{\sigma^{-1}} \mid \sigma \in \Omega, x_{\sigma(1)} \geq \dots \geq x_{\sigma(n)}).$$

In the case of IOWA, an order-introducing n -tuple \mathbf{u} , which is fixed and equal for all inputs \mathbf{x} , is considered. Similarly, it can be defined as $\text{IOWA}_{\mathbf{w}, \mathbf{u}}(\mathbf{x}) = \text{WAM}_{\mathbf{w}, \mathbf{u}}(\mathbf{x})$, where $\mathbf{w}_{\mathbf{u}} = \text{AM}(\mathbf{w}_{\sigma^{-1}} \mid \sigma \in \Omega, u_{\sigma(1)} \geq \dots \geq u_{\sigma(n)}).$

Let us consider an n -tuple $\mathbf{x} \in [0, 1]^n$. Our approach draws inspiration from previous representations. For any weighting vector \mathbf{w} and an n -ary vector $\mathbf{b}(\mathbf{x}) \in [0, 1]^n$, the pair $(\mathbf{w}, \mathbf{b}(\mathbf{x}))$ can be used to define a new weighting vector $\mathbf{w}_{\mathbf{b}(\mathbf{x})} = \text{AM}(\mathbf{w}_{\sigma^{-1}} \mid \sigma \in \Omega, b(\mathbf{x})_{\sigma(1)} \geq \dots \geq b(\mathbf{x})_{\sigma(n)}).$

In general, for any $\mathbf{x} \in [0, 1]^n$, an n -tuple $\mathbf{b}(\mathbf{x}) \in [0, 1]^n$ can be defined, and new generalisations of WAMs, OWAs and IOWAs are introduced. To demonstrate the proposed approach, several examples are presented, including related figures.

Keywords: IOWA operator · OWA operator · Weighted arithmetical mean

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Keynote speech:

Entropies, measures of uncertain and imprecise information

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Abstract: After the introduction of entropies in a probabilistic framework, this concept has been extended to the evaluation of the fuzziness or non-probabilistic uncertainty to evaluate the imprecision or imperfection associated with the observation of objects or events, for instance in the case of intuitionistic or interval-valued fuzzy sets. We focus on the common properties and the differences between such quantities, mainly the monotonicity with respect to a refinement of the information on these objects or events, considering diverse views of this refinement, subsuming or weakening the classic properties of additivity and recursivity. The utilisation of such entropies in artificial intelligence is discussed.

Analytical Properties of General Unary Hypotheses Automaton Quantifiers

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Abstract: In this paper, we explore the analytical properties of general unary hypotheses automaton quantifiers. Specifically, we present two distinct representations of implicational quantifiers and a method for generalizing these representations to double implicational and equivalence quantifiers. Furthermore, we examine selected properties of implicational quantifiers in relation to fuzzy set operations and their associated properties.

Keywords: Fuzzy Quantifiers · Implicational Quantifiers · GUHA method · Fuzzy Logic · Association Rules

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Linear spline copulas

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Abstract: One of the most important classes of 2-dimensional aggregation functions are 2-copulas (copulas, in short). Recall that a function: $C : [0, 1]^2 \rightarrow [0, 1]$ is a copula whenever it has a neutral element $e = 1$, an annihilator $a = 0$, and, for each rectangle $R = [x_1, x_2] \times [y_1, y_2] \subseteq [0, 1]^2$ the C-volume $V_C(R) = C(x_1, y_1) + C(x_2, y_2) - C(x_1, y_2) - C(x_2, y_1)$ is non-negative. Note that each copula C is increasing (and thus an aggregation function) and 1-Lipschitz. For more details we recommend books [1,2,3]. Linear splines play an important role in the approximation of real functions. This fact inspired us to study and discuss copulas, which are linear splines. Observe that a linear spline copula C is affine on simplices S_1, \dots, S_k covering the unit square $[0, 1]^2$ and they are overlapping on segments, possibly. Each such copula is singular with support on some finitely many segments with masses uniformly distributed over those segments. Note that segment support does not generally have to lead to a linear spline copula. As an example, consider a singular copula C given by

$$C(x, y) = \begin{cases} \min(x^2, y^2) & \text{if } x + y \leq 1 \\ \max(x^2, y^2) - |x - y| & \text{else.} \end{cases}$$

The support of this copula is the same as the support of linear spline copulas $\lambda M + (1 - \lambda)W$, $\lambda \in]0, 1[$. We will discuss constructions of the linear spline copulas (e.g., Archimedean case, different types of ordinal sums, shuffles, convex combinations, etc.). Several examples will be given, including illustrative figures. Finally, observe that linear spline copulas are dense in the class of all copulas.

Keywords: Aggregation function · Copula · Linear spline.

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Some notes on permutation-equivariant decomposition integrals

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Abstract: Non-linear integration is a powerful tool in mathematics with applications in multicriteria decision-making and image processing. While symmetric aggregation functions simplify computations [2], they often fail to capture real-world complexities. To address this, we propose a symmetry framework based on permutations, called permutation-equivariant functions, within the framework of aggregation functions to better reflect relationships between criteria or values.

Since decomposition integrals [1] are aggregation functions, we can examine the property of permutation-equivariantness in their framework. Decomposition integrals are built upon decomposition systems \mathcal{H} which are non-empty sets of collections, where a collection is any non-empty subset of $2^X \setminus \{\emptyset\}$, where $X = \{1, 2, \dots, n\}$ for some $n \in \mathbb{N}$. For a decomposition system \mathcal{H} and a monotone measure μ , i.e., a grounded non-decreasing set function on 2^X , we define the decomposition integral by

$$\text{dec}_{\mathcal{H}}^{\mu}(\mathbf{x}) = \bigvee_{\mathcal{D} \in \mathcal{H}} \bigvee \left\{ \sum_{A \in \mathcal{D}} \alpha_A \mu(A) : \sum_{A \in \mathcal{D}} \alpha_A \mathbf{1}_A \leq \mathbf{x}, \alpha_A \geq 0 \text{ for } A \in \mathcal{D} \right\},$$

for all inputs $\mathbf{x} \in [0, \infty]^n$, where $\mathbf{1}_A$ denotes the indicator vector of the set $A \subseteq X$.

In the work, we establish conditions on decomposition systems and monotone measures under which the decomposition integrals are permutation-equivariant aggregation functions. Furthermore, potential applications of such integrals are proposed and explored in the context of image processing, particularly addressing the problem of edge detection.

Keywords: permutation-equivariant functions · aggregation functions · decomposition integrals

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Vertical sum of concept lattices via bonds

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Abstract: Formal concept analysis provides a significant procedure to aggregate (relational) datasets based on bonds, without altering the original information. This paper studies this procedure for aggregating fuzzy formal contexts, with a view to obtaining a vertical sum of the corresponding concept lattices.

Keywords: Formal concept analysis · bonds · multi-adjoint framework · vertical sum

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Bipolar max-product fuzzy relation equations with floor negation

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Abstract: This paper introduces the concept of floor negation, which is a modification of a negation operator so that it only applies below a certain threshold and takes the value 0 above that threshold. Then, bipolar max-product fuzzy relation equations with floor negation are studied. In particular, we present a simple characterization of their solvability and we characterize the existence of maximal and minimal solutions.

Keywords: Bipolar fuzzy relation equation · Max-product composition · Involutive negation

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New Simplification Rules for Databases with Positive and Negative Attributes

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Abstract: This keywork summarises the findings in the recently published paper “D. López-Rodríguez, M. Ojeda-Hernández, and C. Bejines. *New Simplification Rules for Databases with Positive and Negative Attributes*. In: Mathematics 13.2 (2025)”. This paper builds upon prior work to propose new logical equivalences for managing mixed implications in databases, introducing an iterative method for simplification that delivers concise and efficient results.

Formal Concept Analysis (FCA) revolves around the concept of a formal context, denoted as $\mathbb{K} = (G, M, I)$, where G is a set of objects, M a set of attributes, and $I \subseteq G \times M$ the incidence relation specifying which objects have which attributes. In this work, a *closed-world* assumption is adopted, meaning that if $(g, m) \notin I$, it is explicitly known that object g does not possess attribute m . Mixed attributes are introduced by considering both M , the set of positive attributes, and $\bar{M} = \{\bar{m} \mid m \in M\}$, the set of negative attributes, where \bar{m} indicates the absence of m . Derivation operators $\uparrow: 2^G \rightarrow 2^{M \cup \bar{M}}$ and $\downarrow: 2^{M \cup \bar{M}} \rightarrow 2^G$ are defined as $X^\uparrow = \{m \in M \mid gIm \text{ for all } g \in X\} \cup \{\bar{m} \in \bar{M} \mid gIm \text{ for all } g \in X\}$ and $Y^\downarrow = \{g \in G \mid gIm \text{ for all } m \in Y\} \cap \{g \in G \mid g\not Im \text{ for all } \bar{m} \in Y\}$.

Mixed implications, under this point of view, are expressions $A \rightarrow B$ where $A, B \subseteq M \cup \bar{M}$. An implication $A \rightarrow B$ is valid in \mathbb{K} if $B \subseteq A^{\uparrow}$. The Simplification Logic for Mixed Attributes [1] provides axioms and inference rules to reduce redundancy in these implications. Main rules include Reflexivity ($A \rightarrow A$), Augmentation ($A \rightarrow B \vdash AC \rightarrow BC$), Simplification ($A \rightarrow B, C \rightarrow D \vdash A(C \setminus B) \rightarrow D$), Key ($A \rightarrow b \vdash A\bar{b} \rightarrow M \cup \bar{M}$) and Reduction ($Ab \rightarrow C, A\bar{b} \rightarrow C \vdash A \rightarrow C$). These rules enable the construction of simplified and efficient implicational systems that preserve the information encoded in the formal context while reducing computational complexity.

In [3], the authors presented a collection of equivalence rules to simplify mixed implications, in pursue of less redundant implicational systems. The present work provides an additional set of equivalence rules which enables a more concise representation of the implicit knowledge.

Theorem 1 ([2]). *Let $\mathbb{K} = (G, M, I)$ be a formal context and let $A, B, C, D \subseteq M \cup \bar{M}$.*

- If there exists $x \in A \cap \bar{C}$, $y \in B \cap \bar{C}$ with $A \setminus x \subseteq C \setminus \{\bar{x}\bar{y}\}$. Then,

$$\{A \rightarrow B, C \rightarrow D\} \equiv \{A \rightarrow B, C \setminus \bar{x} \rightarrow D\}.$$
- If there exist $x \in A \cap C$, $b \in B$ and $d \in D$ such that $(A \setminus x)\bar{b} \subseteq (C \setminus x)\bar{d}$, then

$$\{A \rightarrow B, C \rightarrow D\} \equiv \{A \rightarrow B, C \rightarrow D \setminus d\}.$$
- If there exist $x \in A$, $y \in B$ such that $(A \setminus x)\bar{y} \subseteq C$, then

$$\{A \rightarrow B, C \rightarrow D\} \equiv \{A \rightarrow B, C \setminus \bar{x} \rightarrow D \setminus \bar{x}\}.$$
- If there exist $x \in A$, $y \in B$ such that $(A \setminus x)\bar{y} \subseteq CD$, then

$$\{A \rightarrow B, C \rightarrow D\} \equiv \{A \rightarrow B, C \rightarrow D \setminus \bar{x}\}.$$

Note that, in all cases, the total number of attributes in the implicational system is reduced as these rules are applied. Thus, their iterative application defines an algorithm for the simplification of mixed implicational systems. The effectiveness of said algorithm is compared to the one that utilizes the rules presented in [3]. Results show that the proposed algorithm [2] achieves the most concise systems compared to previous approaches. Due to the trade-off between simplification and computational cost, the result of applying only one and two iterations is also compared to the computation of the whole simplification algorithm.

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Formal concept analysis and its fuzzy extensions for Light-Bot game solutions

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Abstract: Formal Concept Analysis is a set of methods rooted in mathematical lattice theory which organize data based on objects and their common attributes [1]. Variants of Formal Concept Analysis, including fuzzy extensions, offer promising methods to analyze, visualize, and categorize student solutions. These methods help group solutions into overlapping biclusters (formal concepts) and reveal relationships between attributes [2,3,4,5,6,7,8,9,10,11,12,13]. Recently, we proposed extensions working with formal contexts of heterogeneous data structures [14,15,16,17].

Computational thinking can be understood as a collection of skills involving cognitive or thought processes, such as logical reasoning, used to solve problems and gain a deeper understanding of procedures, either with or without the use of a computer [18,19].

In [20], we presented the solution implemented to help educators explore connections between features of students' solution to problem-solving and computational thinking. The framework included tools for visualization and analysis, enabling the identification of underlying patterns in student solutions. These insights can assist teachers in predicting specific student behaviors, addressing potential errors, and proactively intervening to address misconceptions or improve learning outcomes. Using a dataset of 64 student solutions, we constructed formal contexts based on extracted attributes. FCA was employed to create concept lattices from binary contexts and derive both standard and fuzzy attribute implications to uncover attribute dependencies.

In this paper, we will present extended results on whether Formal Concept Analysis can effectively describe and analyze the diverse solutions and attribute relationships in the educational and computational thinking game Light-Bot.

The analysis highlights several key findings. First, the concept lattice for binary contexts identifies the largest biclusters, representing groups of similar solutions. Second, attribute implications provide insights into patterns, such as solutions with a higher number of executed commands. Finally, fuzzy attribute

implications reveal characteristics of solutions with inefficiencies, such as unnecessary commands, moves outside the game area, or the use of indirect recursion. These results offer a detailed understanding of student behavior and solution strategies in computational problem-solving tasks.

Keywords: Formal Concept Analysis · Fuzzy extensions · Ligh-bot game · Computational thinking

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Composition as a fuzzy conjunction between indexes of inclusion

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Abstract: The φ -index of inclusion was presented originally in [6] as a novel approach to model the inclusion between fuzzy sets and it was extended to general L -fuzzy sets in [3]. The main difference with respect to the existing approaches in the literature is that the inclusion between fuzzy sets is represented by mappings, instead of by values in the lattice of truth degrees.

Since this seminal approach [6], many different results have been obtained to support its use as a good generalization in the fuzzy setting of the classical inclusion of sets. For example, the φ -index of inclusion is in accordance with most of the pivotal axiomatic approaches of inclusion measures between fuzzy sets [2]; the φ -index of inclusion and the φ -index of contradiction [1] can be used to define a square of opposition, in the line of the Aristotelian one [5]; the mapping that defines φ -index of inclusion can be used in a modus ponens inference which is optimal with respect to the set of modus ponens defined from residuated pairs [4].

In this talk we go one step further in the research of the φ -index of inclusion showing that the composition of functions can be used as a suitable conjunction in the set of indexes of inclusion. It is worth to point out two very interesting results. The first one. Let \mathcal{G} be the set of mappings $f: [0, 1] \rightarrow [0, 1]$ satisfying that $f(x) \leq x$ and there exists $g: [0, 1] \rightarrow [0, 1]$ such that (f, g) is an adjoint pair. Then, the composition is a \mathcal{G} -fuzzy conjunction associated to two residuated implications denoted \searrow_{\circ} and \nearrow° ; i.e., $(\mathcal{G}, \circ, \searrow_{\circ}, \nearrow^{\circ}, id)$ is a birresiduated lattice.

The second result is a representation theorem obtained for biresiduated lattices (which includes the standard residuated lattices) in $[0, 1]$.

Theorem 2. *Let $([0, 1], *, \nearrow, \searrow, 1)$ a biresiduated lattice, then there exists $\Theta \subseteq \mathcal{G}$ such that $([0, 1], *, \nearrow, \searrow, 1)$ is isomorphic to $(\Theta, \circ, \searrow_{\circ}, \nearrow^{\circ}, id)$.*

In other words, we can identify each biresiduated lattice in $[0, 1]$ with a sub-biresiduated lattice of $(\mathcal{G}, \circ, \searrow_{\circ}, \nearrow^{\circ}, id)$. In particular, any t -norm can be identified with the composition \circ in a subset of \mathcal{G} .

Keywords: Galois Connections · Inclusion measure · Residuated lattices

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Towards the use of the φ -index of inclusion for fuzzy reasoning

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Abstract: In this talk we present a novel Fuzzy Inference System (FIS) based on the f -inclusion [3]. The f -inclusion is a generalization of the notion of inclusion in Fuzzy Set Theory (FST) in accordance with the pivotal axiomatic approach of Sinha-Dougherty [2]. The main difference with respect to other approaches dealing with inclusions in FST is that the inclusion is represented by functions instead of by values in $[0, 1]$. This functional representation allows to model a Generalized Modus Ponens based on the f -inclusion [1], which is the core of the inference engine of the proposed FIS.

The proposed FIS is supported on a formal semantics where the notions of models and logical consequence can be formally defined. As a result, the FIS can be considered a formal fuzzy logic systems, which provides reliability and theoretical grounding. The obtained FIS can be compared with the so-called Relational FIS, but there are two main differences:

- The RFIS require a fuzzy relation to link the universes of X and Y and to compute the output. In our approach, the connection between universes is left to interpretations and models. This simplifies the computation of the output in our FIS.
- The RFIS require to fix a pair of operators in advance; in general an implication operator and a conjunction operator. However, we do not need to fix operators in advance, the inference is done directly by a mapping that represents the inclusion between fuzzy sets.

Under our opinion, these two differences make of approach more accesible to the applied audience.

Keywords: Galois Connections · Inclusion measure · φ -index of inclusion · Fuzzy sets · Relational Fuzzy Inference Systems

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